## Astronomy C

UT Invitational, Fall 2017
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## Exploring the World of Science

## Competitors:

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## School Name:

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Team Number: $\qquad$

This test contains 4 parts, each of which is worth 25 points. As always, you'll have 50 minutes to complete the test. You may separate the pages; be sure to put your team number at the top of every page. Don't feel obligated to write in complete sentences; your priority is to get all your ideas on paper quickly. I think this might be a long test.

No wifi allowed, blah blah, hopefully you know the rules by now. If you have questions, I'll try to clarify, but I won't give hints. Good Luck, Have Fun!

And always remember: The Eyes of Texas Are Upon You!

| Question: | 1 | 2 | 3 | 4 | Total |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Points: | 25 | 25 | 25 | 25 | 100 |
| Score: |  |  |  |  |  |

1. (25 points) Betelgeuse is a semiregular variable star in Orion. Its variability is of interest to astronomers.
(a) (6 points) Complete the following information about Betelgeuse.

| i. (2 points) Bayer designation: |
| :--- |
| ii. (2 points) Spectral type: $\quad \alpha$ Orionis |

iii. (2 points) Approx. avg. radius $\left(R_{\odot}\right): \quad 1000 \pm 200$
(b) (4 points) Identify Orion and indicate the location of Betelgeuse. (Tiebreaker: label the other stars.) Is Betelgeuse visible from North America?


Solution: Orion is the third constellation (1); Betelgeuse is the rightmost star in Orion in the picture shown (2). It is visible from NA (1) during the winter.
(c) (5 points) Astronomers notice that stars like Betelgeuse undergo (somewhat) periodic changes in apparent brightness. What astrophysical process are these stars undergoing that produces this change in brightness?

## Solution: Stellar pulsation

(d) (10 points) You, an amateur astronomer, find the average apparent magnitude of Betelgeuse over the course of several decades to be 0.5 . Then you carefully measure the color of Betelgeuse, which corresponds to an absolute magnitude of +14 on the HR-diagram main sequence. Then you use the distance modulus to find the distance to Betelgeuse, ignoring the effects of extinction ("They're negligible," you claim confidently). Is your calculated distance reasonable? If so, show the calculation. If not, explain what was wrong in the methodology.

Solution: You will arrive at an unreasonable answer (4) because the absolute magnitude of Betelgeuse is not +14 . Observe that although this is the magnitude of a main sequence $M$ type star, Betelgeuse is not on the main sequence (6). Since it is a giant, its absolute magnitude is much brighter.
2. (25 points) Math is fun!...for some people. I'm putting only calculation questions here so that I don't have to write any more calculation questions for the rest of this dumb test. Oh yeah.
(a) (5 points) Recently, there was a bunch of hype about two orbiting neutron stars that collided and produced both detectable light and detectable gravitational waves. Apparently it was a big deal. Anyways, assuming that the neutron stars were 2 solar masses each and 1000 km apart, what was their orbital period? Ignore relativistic effects.

Solution: Using Kepler's 3rd law (1),

$$
P^{2}=a^{3} \frac{4 \pi^{2}}{G(M+m)}=\left(1.00 \times 10^{6}\right)^{3} \frac{4 \pi^{2}}{G * 2 * 2\left(1.989 \times 10^{30}\right)} \approx 0.074
$$

Therefore,

$$
\begin{equation*}
P \approx 0.27 s \tag{4}
\end{equation*}
$$

That's one speedy boi!
(b) (10 points) You've discovered a star! You name it Bob, after your favorite minion from Despicable Me. You calculate the surface flux of Bob to be $6.00 \times 10^{7} \mathrm{~W} \mathrm{~m}^{-2}$.
i. (4 points) At what wavelength does Bob emit the most light?

Solution: Using the Stefan-Boltzmann equation (1),

$$
F_{\text {surf }}=\sigma T^{4}, \sigma=5.67 \times 10^{-8} \mathrm{~W} \mathrm{~m}^{2} \mathrm{~K}^{-4} \Rightarrow T \approx 5703 \mathrm{~K}
$$

Then, using the Wien displacement law (1),

$$
\begin{equation*}
\lambda_{\max } T=b, \quad b=2.90 \times 10^{-3} \mathrm{mK} \Rightarrow \lambda_{\max }=b / 5703 \mathrm{~K} \approx 508 \mathrm{~nm} \tag{2}
\end{equation*}
$$

ii. (6 points) You realize that the peak wavelength is in the green part of the spectrum. Uh oh. You rush back to the telescope, only to confirm with chagrin that Bob appears to be, just like his namesake, more yellow than any other color. Why is Bob not green?

Solution: Although Bob emits more green light than any other color, it still emits light from all frequencies (2). Because of the asymmetry in the blackbody radiation curve (1), Bob emits more yellow and orange light than blue light (2); therefore the "average" color that we see is shifted towards the yellow side of the spectrum, and we observe a yellow-white star (1).
(c) (10 points) Who doesn't love stellar pulsation? One important factor in the pulsation mechanism is the opacity of the gas in the star. The equation for opacity is given by $\Delta I=-\kappa \rho I \Delta s$, where $I$ is light intensity, $\kappa$ is the gas opacity, $\rho$ is the gas density, and $\Delta s$ is the distance the light has traveled through the gas. In turn, the gas opacity is given by $\kappa=c \rho / T^{3.5}$, where $c$ is a positive constant. In some arbitrary star, let's consider two places in its interior. In location $B$, the gas density is twice that of location A, but the light intensity at location B is half that of A. If the light intensity changes at the same rate through the gas at A and B, how much larger/smaller is the temperature at B than at A ?

Solution: Let the rate of change of light intensity at location A be

$$
\begin{equation*}
\frac{\Delta I}{\Delta s}=-\kappa \rho I=-\left(c \rho / T^{3.5}\right) \rho I=\frac{-c \rho^{2} I}{T^{3.5}} \tag{2}
\end{equation*}
$$

Since we know that $I_{b}=I / 2$ and $\rho_{b}=2 \rho$, we conclude

$$
\begin{equation*}
\frac{\Delta I_{b}}{\Delta s_{b}}=\frac{-c(2 \rho)^{2}(I / 2)}{\left(T^{\prime}\right)^{3.5}}=2 \frac{-c \rho^{2} I}{\left(T^{\prime}\right)^{3.5}} \tag{2}
\end{equation*}
$$

Since we are told that the rate of change of light intensity is equal at both A and B , we can equate the two equations to each other:

$$
\begin{equation*}
2 \frac{-c \rho^{2} I}{\left(T^{\prime}\right)^{3.5}}=\frac{-c \rho^{2} I}{T^{3.5}} \tag{2}
\end{equation*}
$$

Simplifying, we arrive at

$$
\begin{equation*}
2=\frac{\left(T^{\prime}\right)^{3.5}}{T^{3.5}}=\left(\frac{T^{\prime}}{T}\right)^{3.5} \Rightarrow \frac{T^{\prime}}{T}=\sqrt[3.5]{2} \approx 1.22 \tag{4}
\end{equation*}
$$

Therefore the temperature at B is about 1.22 times as large as the temperature at A.
3. ( 25 points) AG Carinae is a bright star. It's like... really bright. Super bright, actually. Almost as bright as when you're on a bus at 5 am and you're trying to get some sweet, sweet sleep and then the bus driver turns on the main bus lights.
(a) (3 points) Refer to section 3a on the diagram sheet. Which image(s) represent AG Carinae?

Solution: A, C, E (+1 for every correct, $\mathbf{- 1}$ for every incorrect, minimum 0)
(b) (15 points) Ah, the classic HR Diagram question. Sketch an HR diagram and include the following:
i. (2 points) Label the axes (you may use any conventional axes pair.)
ii. (4 points) Sketch the main sequence. Indicate where the Sun is.
iii. (4 points) Indicate where AG Carinae is.
iv. (5 points) Sketch the evolutionary track of AG Carinae, from main sequence to predicted death (3pt). Label the LBV phase and WR phase (2pt).

(c) (2 points) Which one of the following is most likely responsible for AG Carinae's high rates of mass loss? Choose an answer and very briefly justify.

## A. Eddington Luminosity Limit <br> B. Chandrasekhar Limit <br> C. Dirac's Kinetic Limit <br> D. Rosen's Plasma Limit

Solution: Sample response: The Eddington Limit is the limit at which radiation pressure exceeds gravity. If AG Carinae exceeds this limit, the excess radiation pressure will result in mass loss.
Another possible response: The Chandrasekhar Limit is the maximum mass for white dwarfs; the other two are completely made-up. The only answer left is the Eddington Luminosity Limit.
(d) (5 points) Stars like AG Carinae can often end up as black holes if they're not careful. But these stellar-mass black holes are very different from the supermassive black holes that reside in the center of galaxies. An astronaut could theoretically survive well past the event horizon if they were falling into a supermassive black hole. This is not the case for stellar mass black holes; an astronaut would be absolutely shredded much before entering the black hole. What is the name of this gruesome process? Explain the physical phenomenon that causes this. BONUS: Why is the phenomenon much stronger in smaller-mass black holes, and not in larger ones?

Solution: The process is called spaghettification (2). It's caused by tidal forces, which in turn is caused by the gravity differential between your head and feet. This differential means that your feet are pulled towards the black hole faster than your head is, stretching you until you rip (3).
(BONUS) Smaller mass black holes have much steeper gravitational fields, meaning that the gravity differential between your head and feet is much stronger. The important thing to remember is that spaghettification is not a result of the strength of the gravitational field; it's a result of the magnitude of the differential across your body.

## 4. (25 points) SUPERNOVA!

(a) (3 points) Refer to section 4a in the diagram sheet. What is the name of the event that produced this light curve?

## Solution: SN 1987A

(b) (10 points) The identity of the supernova's progenitor surprised astronomers.
i. (2 points) What was surprising about it?

Solution: It was a blue supergiant (1). Astronomers thought that only red supergiants could undergo a supernova event (1).
ii. (8 points) Explain how the identity of the progenitor influenced the light curve that we observed. How and why was this light curve different from the light curves of other Type II supernovae?

Solution: The peak luminosity was dimmer than other Type II supernovae (3) by about 2.5 magnitudes (1). This is a result of the fact that blue supergiants are much denser than red supergiants (2), so it takes more of the energy generated by SN 1987A to blow the ejecta away (2).
(c) (7 points) Neutrinos from this event were detected several hours before we detected photons. Does this imply that neutrinos travel faster than light? If yes, explain the implications on special relativity. If no, give the correct explanation for the observation.

> Solution: No, it doesn't imply that neutrinos are faster than light (3). During the supernova event, the dense envelope became transparent to neutrinos several hours before it became transparent to photons (4) (since neutrinos interact with matter less than photons).
(d) (5 points) One hypothesis to explain the surprise (in part 4bi) is that the progenitor star had $Z \approx 0$. Considering the location of the progenitor, is the premise of this hypothesis reasonable? Why or why not?

Solution: Yes, the hypothesis is reasonable (1). The supernova ocurred in the Large Magellanic Cloud (2), which has low metallicity (2).

## Diagrams

3a


4a


